Hawking-like radiation from the apparent horizon in an FRW Universe : Quantum Corrections

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Abstract

In this paper we study Hawking-like radiation in a FRW-Universe using semi-classical tunnelling approach and the Hamilton-Jacobi method. Radial null geodesics are used to picture the process as a "tunnelling" of particles from behind the apparent horizon and Hawking -like temperature has been calculated. Quantum corrections have been evaluated in the Hamilton-Jacobi Method by solving Klein-Gordan wave equation and the temperature agrees at the semiclassical level. Also it is found that Hamilton-Jacobi formalism does not depend on the choice of the coordinate system. Finally, leading order corrections to entropy has been calculated.

Keywords : Hawking-like Temperature, FRW Universe, Tunnelling of Particles.

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1 Introduction

In recent years there are lots of works [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] in universal thermodynamics with a view to establish the equivalence between the Friedmann equations and the first law of thermodynamics. These studies are mostly concentrated to FRW model of the Universe (For universal thermodynamics in LTB model see ref [12]). In these works universe bounded by the apparent horizon is chosen as a thermodynamical object and the entropy and temperature of the boundary are assumed to be

$$S = \frac{A}{4G} \quad , \quad T = \frac{1}{2\pi R_A}$$

where A and R_A are the area and radius of the apparent horizon respectively. Jacobson [13] first formulated the Einstein field equations using the fundamental Clausius relation $\partial Q = TdS$. Subsequently, recent progress of black hole thermodynamics has been applied to the universal thermodynamics [14, 15, 6, 17, 18].

To study the Hawking radiation there are two basic approaches − Firstly, the tunnelling approach of Parikh and Wilczek [19, 20, 21] and secondly the standard Hamilton-Jacobi (HJ) method (also known as complex path integral formalism) [22, 23]. Due to simplicity of its calculation there are lot of works [24, 25, 26, 27, 28] in this method. Recently these approaches has been applied to universal thermodynamics. Hawking-like temperature is calculated using both the methods at the apparant horizon of the FRW universe [29, 30, 31, 32, 33, 34] (see also ref [35]) for LTB model of the universe). Initially there was a problem of factor 'two' [31, 32, 33, 34] in the Hawking-like temperature and it was resolved subsequently considering contribution to the probability from the time co-ordinate upon crossing the horizon [36, 37, 38].

In the present work, we make an exhaustive study of the FRW universal thermodynamics using both the methods. We review the radial null geodesic method using Kodama vector in section 2. Section 3 deals with

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HJ approach in which full quantum corrections have been evaluated. Semiclassical Hawking-like temperature is also calculated using Painleve type coordinates by HJ formulation in section 4 and the temperature agrees with the previous results. Finally, the paper ends with a brief conclusion in section 5.

2 Hawking-like temperature by radial null geodesic method:An Overview

In recent years the popular approach of evaluating Hawking radiation is the radial null geodesic method [19, 20, 21](known as semi classical tunnelling analysis). The method is simple compared to HJ method but it has some limitations, namely

(i) The method is applicable only for massless particles

(ii) one has to use only Painleve type coordinates to avoid singularity at the horizon

(iii) there is a discrepancy of factor two in this method

(iv) there is no general method to include quantum effects(back reaction).

The basic idea in this semiclassical approximation (WKB approximation) is that the emission rate for the s-wave emission of a massless particle can be related to the imaginary part of the action of a system. In the s-wave approximation, particles can be viewed as a massless shell moving along radial null geodesic. Now compared to static black hole cases, there is one basic difference as in the present case the metric coefficients depend on both radius and time . Hence there is no longer time translation Killing vector field. We shall have to use the Kodama vector [39] which is time like inside the horizon and the associate energy.

The homogeneous and isotropic model of the universe described by the FRW metric as

$$ds^2 = -dt^2 + \frac{a^2(t)}{1-kr^2}dr^2 + a^2(t)r^2 d\Omega_2^2 \tag{1}$$

Here the FRW coordinate $(t,\ r,\ \theta,\ \phi)$ is an orthogonal co-moving coordinate with $'t'$ the co-moving time, corresponding to a co-moving observer and k is the intrinsic spatial curvature.

For the above FRW model of the universe if we make a change of the radial coordinate : $r \to R$ where $R = ar$ is known as area radius then the above standard FRW metric become Pinleve-Gustrand-like metric [40] as follows

$$ds^2 = -\frac{1-\frac{R^2}{R_A^2}}{1-\frac{kR^2}{a^2}}dt^2 - \frac{2HR}{1-\frac{kR^2}{a^2}}dtdR + \frac{dR^2}{1-\frac{kR^2}{a^2}} + R^2 d\Omega_2^2 \tag{2}$$

Here $R_A = \frac{1}{\sqrt{H^2+\frac{k}{a^2}}}$ is the radius of the apparent horizon.

In the present case for the metric (2) the Kodama vector and the corresponding energy are

$$\left.\begin{array}{c} k^\mu = \left(\sqrt{1-\frac{kR^2}{a^2}},\ 0,\ 0,\ 0\right) \\ \\ and \\ \\ \omega = -\sqrt{1-\frac{kR^2}{a^2}}\frac{\partial S}{\partial t} \end{array}\right\} \tag{3}$$

Thus $\frac{\omega}{\sqrt{1-\frac{kR^2}{a^2}}}$ is the energy of the particle as measured by an observer with the Kodama vector.

The differential equation for the radial null geodesic (i.e., $ds^2 = 0 = d\Omega_2^2$) has the form :

$$\frac{dR}{dt} = HR \pm \sqrt{H^2R^2 + 1 - \frac{R^2}{R_A^2}} \tag{4}$$

where $+/-$ sign are associated with outgoing /ingoing null geodesic, with the assumption that $'t'$ increases towards future. As we are interested in the imaginary part of the action corresponding to the tunnelling process through a barrier (the classically forbidden region) so according to Parikh-Wilczek [19]

$$ImS = Im\int_{R_{in}}^{R_{out}} p_R dR = Im\int_{R_{in}}^{R_{out}}\int_0^{p_R} dp'_R dR = Im\int_{R_{in}}^{R_{out}}\int_0^E \frac{dH'}{\dot{R}}dR \tag{5}$$

where in the last step we have used the Hamiltonian equation

$$\dot{R} = \frac{\partial H}{\partial p_R} = \frac{dH}{dp_R}\bigg|_R$$

and here p_R is the radial momentum $R_{in},\ R_{out}$ are positions very close to horizon with R_{in} the initial position and R_{out}, a classical turning point. Using the value of $\dot{R}$ from equation (4) into (5) we get

$$ImS = Im \int_{R_{in}}^{R_{out}} dR \int \frac{dH'}{\dot{R}}$$

$$= Im \int_{R_{in}}^{R_{out}} \frac{dR}{\dot{R}} \frac{\omega}{\sqrt{1 + \frac{kR^2}{a^2}}}$$

$$= -\omega \int_{R_{in}}^{R_{out}} \frac{dR}{\sqrt{1 + \frac{kR^2}{a^2}} \left\{ \sqrt{H^2R^2 + 1 - \frac{R^2}{R_A^2}} - HR \right\}}$$

$$= \pi R_A \omega \tag{6}$$

where integration over the Hamiltonian 'H' gives the energy of the particle as $\frac{\omega}{\sqrt{1-\frac{k^2}{a^2}}}$ as measured by an observer with Kodama vector.

Now comparing the tunnelling probability

$$\Gamma \sim \exp\left\{-\frac{2}{\hbar} ImS\right\} \tag{7}$$

with the Boltzmann factor $\exp\left\{-\frac{\omega}{T}\right\}$ we have the temperature associated with the apparent horizon of the FRW universe as

$$T_A = \frac{\hbar}{2\pi R_A} \tag{8}$$

This is the semi-classical Hawking-like temperature of the FRW universe for tunnelling of massless particles across the apparent horizon.

3 Hamilton-Jacobi method : Quantum prescription

This section deals with tunnelling of massless particle beyond semiclassical approximation by Hamilton-Jacobi (HJ) method.

We start with the Klein-Gordan(KG) equation for a scalar field ϕ describing a massless scalar particle of the form

$$-\frac{\hbar^2}{\sqrt{-g}} \partial_\mu \left(g^{\mu\nu}\sqrt{-g}\partial_\nu\right)\phi = 0 \tag{9}$$

The explicit form of the KG equation in the background of the above FRW metric (1) is given by

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\left(1 - kr^2\right)}{a^2}\frac{\partial^2\phi}{\partial r^2} + H\frac{\partial\phi}{\partial t} + \frac{kr}{a^2}\frac{\partial\phi}{\partial r} = 0 \tag{10}$$

It should be mentioned here that due to spherical symmetry of the FRW space time and consideration of the radial trajectories only we have considered $(t - r)$-sector in the space time given by equation (1), i.e., 2D hyperplane $(t,\ r)$. Now substituting the standard ansatz for the semiclassical wave function

$$\phi(r,\ t) = \exp\left\{\frac{i}{\hbar}S(r,\ t)\right\} \tag{11}$$

into the wave equation (10), the differential equation for the action S becomes

$$\left(\frac{\partial S}{\partial t}\right)^2-\left(\frac{1-kr^2}{a^2}\right)\left(\frac{\partial S}{\partial r}\right)^2-i\hbar\left[\frac{\partial^2 S}{\partial t^2}\left(\frac{1-kr^2}{a^2}\right)\frac{\partial^2 S}{\partial r^2}+H\frac{\partial S}{\partial t}+\frac{kr}{a^2}\frac{\partial S}{\partial r}\right]=0 \tag{12}$$

As a first step to solve this partial differential equation(p.d.e.) we expand the action in powers of Planck constant $\hbar$ as

$$S(r,\ t)=S_0(r,\ t)+\Sigma_k\hbar^k S_k(r,\ t) \tag{13}$$

with k, a positive integer. Here terms of the order of Planck's constant and its higher powers are considered as quantum corrections over the semiclassical action S_0. Now substituting this ansatz for S in the differential equation (12) and equating different powers of $\hbar$ on both sides we obtain the following set of partial differential equations :

$$\hbar^0\quad:\quad\left(\frac{\partial S_0}{\partial t}\right)^2-\left(\frac{1-kr^2}{a^2}\right)\left(\frac{\partial S_0}{\partial r}\right)^2=0 \tag{14}$$

$$\hbar^1\ :\ 2\frac{\partial S_0}{\partial t}\frac{\partial S_1}{\partial t}-2\left(\frac{1-kr^2}{a^2}\right)\frac{\partial S_0}{\partial r}\frac{\partial S_1}{\partial t}-i\hbar\left[\frac{\partial^2 S_0}{\partial t^2}-\left(\frac{1-kr^2}{a^2}\right)\frac{\partial^2 S_0}{\partial r^2}+H\frac{\partial S_0}{\partial t}+\frac{kr}{a^2}\frac{\partial S_0}{\partial r}\right]=0 \tag{15}$$

$$\hbar^2\ :\ \left(\frac{\partial S_1}{\partial t}\right)^2+2\frac{\partial S_0}{\partial t}\frac{\partial S_2}{\partial t}-\left(\frac{1-kr^2}{a^2}\right)\left\{\left(\frac{\partial S_1}{\partial r}\right)^2+2\frac{\partial S_0}{\partial r}\frac{\partial S_2}{\partial r}\right\}-i\hbar\left[\frac{\partial^2 S_1}{\partial t^2}-\left(\frac{1-kr^2}{a^2}\right)\frac{\partial^2 S_1}{\partial r^2}+H\frac{\partial S_1}{\partial t}+\frac{kr}{a^2}\frac{\partial S_1}{\partial r}\right]=0 \tag{16}$$

....and so on.

Apparently, different order p.d.e.s are very complicated but fortunately there will be lot of simplifications if in the p.d.e. corresponding to order $\hbar^k$, all previous p.d.e.s are used and finally we obtain identical p.d.e.s namely

$$\hbar^k\quad:\quad\left(\frac{\partial S_k}{\partial t}\right)^2-\left(\frac{1-kr^2}{a^2}\right)\left(\frac{\partial S_k}{\partial r}\right)^2=0\qquad k=0,1,2,...... \tag{17}$$

We see that different order quantum corrections satisfy identical differential equations as the semiclassical action S_0, so the correction terms are not independent, rather proportional to S_0 (i.e., $S_k\propto S_0\quad for\ all\ k$). To determine these proportionality constants we shall use dimension analysis. Since S_0 has the dimension $\hbar$ so the proportionality constant d_k for S_k has the dimension $\hbar^{-k}$. But in standard units namely $G=c=k_B=1$, the Planck's constant $\hbar$ is of the order M_p^2 (M_p=Planck's mass) and hence d_k has the dimension M^{-2k}, where M is identified as the mass of the universe. Thus the series expansion (13) can be written in terms of S_0 as

$$S(r,\ t)=S_0(r,\ t)\left[1+\Sigma_k\alpha_k\left(\frac{\hbar}{M^2}\right)^k\right] \tag{18}$$

with α_k as dimensionless constant parameters.

Now, a complete solution for S requires solution of S_0 satisfying the p.d.e (14). Since the metric (1) is non-static so there is no time like Killing vector in the dynamical FRW space-time. However, Kodama vector [39] has similar role in FRW spacetime as the timelike killing vector does in the stationary black hole space time. Inside the apparent horizon Kodama vector is a time like vector and there is a conserved energy of a particle moving in the time like killing vector in the stationary black hole space time.

The Kodama vector for the FRW metric is given by

$$K^\mu=\left(\sqrt{1-kr^2},\ \ -Hr\sqrt{1-kr^2},\ 0,\ 0\right) \tag{19}$$

with associated conserved energy of the particle

$$\omega=-\sqrt{1-kr^2}\frac{\partial S_0}{\partial t}+Hr\sqrt{1-kr^2}\frac{\partial S_0}{\partial r} \tag{20}$$

Solving for $\frac{\partial S_0}{\partial t}$ and $\frac{\partial S_0}{\partial r}$ using equations (14) and (20) we get

$$S_0=-\int\frac{\omega dt}{\left\{\sqrt{1-kr^2}-Har\right\}}\mp a\omega\int\frac{dr}{\sqrt{1-kr^2}\left\{\sqrt{1-kr^2}-Har\right\}} \tag{21}$$

where $-/+$ sign corresponds to ingoing/outgoing scalar particle for which the wave functions have the expressions

$$\psi_{in} = \exp\left[-\frac{i}{\hbar}\left\{1+\Sigma_k\alpha_k\left(\frac{\hbar}{M^2}\right)^k\right\}\left\{\int\frac{\omega dt}{\left\{\sqrt{1-kr^2}-Har\right\}}+a\omega\int\frac{dr}{\sqrt{1-kr^2}\left\{\sqrt{1-kr^2}-Har\right\}}\right\}\right] \tag{22}$$

and

$$\psi_{in} = \exp\left[-\frac{i}{\hbar}\left\{1+\Sigma_k\alpha_k\left(\frac{\hbar}{M^2}\right)^k\right\}\left\{\int\frac{\omega dt}{\left\{\sqrt{1-kr^2}-Har\right\}}-a\omega\int\frac{dr}{\sqrt{1-kr^2}\left\{\sqrt{1-kr^2}-Har\right\}}\right\}\right] \tag{23}$$

Now, across the horizon the metric coefficients in the $(r,\ t)$ sector alter their sign, so the above time integration may have imaginary part and have contribution to the probabilities for the ingoing and the outgoing particles. As a result the probabilities are given by

$$P_{in} = |\phi_{in}|^2 = \exp\left[\frac{2}{\hbar}\left\{1+\Sigma_k\alpha_k\left(\frac{\hbar}{M^2}\right)^k\right\}\left\{Im\int\frac{\omega dt}{\left\{\sqrt{1-kr^2}-Har\right\}}+Im\ a\omega\int\frac{dr}{\sqrt{1-kr^2}\left\{\sqrt{1-kr^2}-Har\right\}}\right\}\right] \tag{24}$$

and

$$P_{out} = |\phi_{out}|^2 = \exp\left[\frac{2}{\hbar}\left\{1+\Sigma_k\alpha_k\left(\frac{\hbar}{M^2}\right)^k\right\}\left\{Im\int\frac{\omega dt}{\left\{\sqrt{1-kr^2}-Har\right\}}+Im\ a\omega\int\frac{dr}{\sqrt{1-kr^2}\left\{\sqrt{1-kr^2}-Har\right\}}\right\}\right] \tag{25}$$

But in the classical limit ($\hbar \to 0$) the outgoing probability has to be unity as then there will be no absorber and everything will go out [26] and hence from (25) we get

$$Im\int\frac{\omega dt}{\left\{\sqrt{1-kr^2}-Har\right\}} = Ima\omega\int\frac{dr}{\sqrt{1-kr^2}\left\{\sqrt{1-kr^2}-Har\right\}} \tag{26}$$

So P_{in} will have the simplified form

$$\begin{aligned} P_{in} &= \exp\left[\frac{4a\omega}{\hbar}\left\{1+\Sigma_k\alpha_k\left(\frac{\hbar}{M^2}\right)^k\right\}Im\int\frac{dr}{\sqrt{1-kr^2}\left\{\sqrt{1-kr^2}-Har\right\}}\right] \\ &= \exp\left[\frac{2\omega}{\hbar}\left\{1+\Sigma_k\alpha_k\left(\frac{\hbar}{M^2}\right)^k\right\}\pi R_A\right] \end{aligned} \tag{27}$$

Hence from the principle of $'detailed\ balance'$, i.e.,

$$P_{out} = \exp\left\{-\frac{\omega}{T_h}\right\}P_{in}$$

we have

$$T_h = \left\{1+\Sigma_k\alpha_k\left(\frac{\hbar}{M^2}\right)^k\right\}^{-1}\frac{1}{2\pi R_A} = \left\{1+\Sigma_k\alpha_k\left(\frac{\hbar}{M^2}\right)^k\right\}^{-1}T_c \tag{28}$$

This is the horizon temperature of the FRW model of the universe after quantum corrections are taken into account. The quantum corrections to this modified temperature is very smilar to that of Banerjee et. al. [26] for general static black holes. Also the modified temperature has arbitrariness due to choice of the parameters α_k in the expansion. Banerjee et. al. [26] has shown for static black holes that different choices of α_k's will lead to different physical interpretation. For future work, we shall attempt to study particles with no-zero mass and examine whether quantum corrections as well as Hawking like temperature depend on the mass term.

4 Hamilton-Jacobi method in painleve type coordinate system for FRW universe

As in the previous section The KG equation for a scalar field ϕ describing a massless scalar particle has the form

$$\hbar^2 \Box \psi = 0$$

$$i.e., \qquad -\frac{\hbar^2}{\sqrt{-g}}\partial_\mu \left(g^{\mu\nu}\sqrt{-g}\partial_\nu\right)\psi = 0$$

where the box operator is evaluated in the back ground of the metric (2) in the $(t,\ r)$ part of the space-time as before.The explicit form of the KG equation is

$$-C\frac{\partial^2\psi}{\partial t^2} + A\frac{\partial^2\psi}{\partial r^2} - 2B\frac{\partial^2\psi}{\partial t\partial r} - \frac{\mu}{2}\left[\frac{\partial}{\partial R}\left(\frac{B}{\mu}\right) + \frac{\partial}{\partial t}\left(\frac{C}{\mu}\right)\right]\frac{\partial\psi}{\partial t} + \frac{\mu}{2}\left[\frac{\partial}{\partial R}\left(\frac{A}{\mu}\right) - \frac{\partial}{\partial t}\left(\frac{B}{\mu}\right)\right]\frac{\partial\psi}{\partial R} = 0 \qquad (29)$$

where $c = \left(1 - \frac{KR^2}{a^2}\right)^{-1}$, $B = \frac{HR}{1-\frac{KR^2}{a^2}}$, $A = \frac{\left(1-\frac{R^2}{R_A^2}\right)}{1-\frac{KR^2}{a^2}}$ and $\mu = AC + B^2$.

If we substitute the standard ansatz for semi classical wave function namely

$$\psi(R,\ t) = \exp\left\{-\frac{i}{\hbar}S(R,\ t)\right\} \qquad (30)$$

then the action S will satisfy the following differential equation

$$C\frac{\partial^2 S}{\partial t^2} - A\frac{\partial^2 S}{\partial r^2} + 2B\frac{\partial S}{\partial t}\frac{\partial S}{\partial R}$$
$$+ i\hbar\left[A\frac{\partial^2 S}{\partial R^2} - C\frac{\partial^2 S}{\partial t^2} - 2B\frac{\partial^2 S}{\partial R\partial t} + \frac{\mu}{2}\left[\left\{\frac{\partial}{\partial R}\left(\frac{A}{\mu}\right) - \frac{\partial}{\partial t}\left(\frac{B}{\mu}\right)\right\}\frac{\partial S}{\partial R} - \left\{\frac{\partial}{\partial R}\left(\frac{B}{\mu}\right) + \frac{\partial}{\partial t}\left(\frac{C}{\mu}\right)\right\}\frac{\partial S}{\partial t}\right]\right] = 0 \qquad (31)$$

In order to incorporate quantum corrections over the semiclassical action, S will expanded in powers of Planck constant $\hbar$ as

$$S(R,t) = S_0(R,t) + \Sigma\hbar^k S_k(R,t) \qquad (32)$$

where S_0 is the semiclassical action and K is a positive integer. Substituting (32) in (31) and equating different powers of $\hbar$ on both sides we have the following differential equations :

$$\hbar^0 \quad : \quad C\left(\frac{\partial S_0}{\partial t}\right)^2 - A\left(\frac{\partial S_0}{\partial R}\right)^2 + 2B\frac{\partial S_0}{\partial R}\frac{\partial S_1}{\partial t} = 0 \qquad (33)$$

$$\hbar^1 \quad : \quad 2C\frac{\partial S_0}{\partial t}\frac{\partial S_1}{\partial t} - 2A\frac{\partial S_0}{\partial R}\frac{\partial S_1}{\partial R} + 2B\left(\frac{\partial S_0}{\partial R}\frac{\partial S_1}{\partial t} + \frac{\partial S_0}{\partial t}\frac{\partial S_1}{\partial R}\right)$$
$$+ i\hbar\left[A\frac{\partial^2 S_0}{\partial R^2} - B\frac{\partial^2 S_0}{\partial t^2} - 2B\frac{\partial^2 S_0}{\partial R\partial t} + \frac{\mu}{2}\left[\left\{\frac{\partial}{\partial R}\left(\frac{A}{\mu}\right) - \frac{\partial}{\partial t}\left(\frac{B}{\mu}\right)\right\}\frac{\partial S_0}{\partial R} - \left\{\frac{\partial}{\partial R}\left(\frac{B}{\mu}\right) + \frac{\partial}{\partial t}\left(\frac{C}{\mu}\right)\right\}\frac{\partial S_0}{\partial t}\right]\right] = 0 \qquad (34)$$

....and so on.

To solve the semi classical action S_0 from equation (7) we first write $\frac{\partial S_0}{\partial R}$ in terms of $\frac{\partial S_0}{\partial t}$ as

$$\frac{\partial S_0}{\partial R} = \frac{HR \pm \sqrt{H^2R^2 + \left(1 - \frac{R^2}{R_A^2}\right)}}{\left(1 - \frac{R^2}{R_A^2}\right)}\frac{\partial S_0}{\partial t} \qquad (35)$$

where $+/-$ sign corresponds to ingoing/outgoing scalar particle. For the Painleve-Gulstrad type coordinates given in equation (2) the energy of the scalar particle can be obtained using Kodama vector [39] as

$$\omega = -\sqrt{1-\frac{kR^2}{a^2}}\frac{\partial S_0}{\partial t} \tag{36}$$

where $\sqrt{1-\frac{kR^2}{a^2}}\frac{\partial}{\partial t}$ is the Kodama vector for the metric (2). Using the value of $\frac{\partial S_0}{\partial t}$ from (36) we obtain $\frac{\partial S_0}{\partial R}$ using (35) and hence

$$S_0(R,\ t) = -\int \frac{\omega dt}{\sqrt{1-\frac{kR^2}{a^2}}} + \omega \int \frac{-HR \mp \sqrt{1-\frac{R^2}{R_A^2}}}{\left(1-\frac{R^2}{R_A^2}\right)\sqrt{1-\frac{kR^2}{a^2}}} dR \tag{37}$$

Then the wave functions denoting ingoing and outgoing solution of the K.G. equation (29) using (30) are of the form

$$\psi_{in} = \exp\left\{-\frac{i}{\hbar}\left(\int \frac{\omega dt}{\sqrt{1-\frac{kR^2}{a^2}}} + \omega \int \frac{HR + \sqrt{1-\frac{R^2}{R_A^2}}}{\left(1-\frac{R^2}{R_A^2}\right)\sqrt{1-\frac{kR^2}{a^2}}} dR\right)\right\} \tag{38}$$

and

$$\psi_{out} = \exp\left\{-\frac{i}{\hbar}\left(\int \frac{\omega dt}{\sqrt{1-\frac{kR^2}{a^2}}} - \omega \int \frac{HR - \sqrt{1-\frac{R^2}{R_A^2}}}{\left(1-\frac{R^2}{R_A^2}\right)\sqrt{1-\frac{kR^2}{a^2}}} dR\right)\right\} \tag{39}$$

As in the previous section due to tunnelling across the horizon the coordinate nature changes,i.e., the sign of the metric coefficients in the $(R,\ t)$-hyperplane are altered so the time integral has an imaginary part and hence it has a contribution to the probabilities [26, 37]. Thus both incoming and out going probabilities are given by

$$P_{in} = |\psi_{in}|^2 = \exp\left\{\frac{2}{\hbar}\left(Im\int \frac{\omega dt}{\sqrt{1-\frac{kR^2}{a^2}}} + \omega Im \int \frac{HR + \sqrt{1-\frac{R^2}{R_A^2}}}{\left(1-\frac{R^2}{R_A^2}\right)\sqrt{1-\frac{kR^2}{a^2}}} dR\right)\right\} \tag{40}$$

and

$$P_{out} = |\psi_{out}|^2 = \exp\left\{\frac{2}{\hbar}\left(Im\int \frac{\omega dt}{\sqrt{1-\frac{kR^2}{a^2}}} - \omega Im \int \frac{HR + \sqrt{1-\frac{R^2}{R_A^2}}}{\left(1-\frac{R^2}{R_A^2}\right)\sqrt{1-\frac{kR^2}{a^2}}} dR\right)\right\} \tag{41}$$

Now some simplifications are possible using the fact that all outgoing particles certaining cross the horizon, i.e., $P_{out} = 1$. So from equation (41)

$$Im\int \frac{\omega dt}{\sqrt{1-\frac{kR^2}{a^2}}} = \omega Im \int \frac{HR - \sqrt{1-\frac{R^2}{R_A^2}}}{\left(1-\frac{R^2}{R_A^2}\right)\sqrt{1-\frac{kR^2}{a^2}}} dR = 0 \tag{42}$$

and so P_{in} simplifies to

$$\begin{aligned} P_{in} = |\phi_{in}|^2 &= \exp\left\{\frac{2\omega}{\hbar} Im \int \frac{HR + \sqrt{1-\frac{R^2}{R_A^2}}}{\left(1-\frac{R^2}{R_A^2}\right)\sqrt{1-\frac{kR^2}{a^2}}} dR\right\} \\ &= \exp\left\{\frac{2\omega}{\hbar}\pi R_A\right\} \end{aligned} \tag{43}$$

Then from the principle of "detailed balance" [22, 23, 41] we write

$$P_{in} = \exp\left\{\frac{\omega}{T_h}\right\} P_{out} = \exp\left\{\frac{\omega}{T_h}\right\} \tag{44}$$

Thus comparing (43) and (44), the temperature of the horizon is given by

$$T_e = \frac{\hbar}{2\pi R_A} \tag{45}$$

Hence we have the same expansion for the temperature as we have obtained in previous sections. Thus choice of coordinate is not matter for the HJ method. Here higher order correction terms can be obtained in a straightforward manner and lead to identical results as in the previous section.

5 Determination of Entropy : Area law

I this section we shall examine the semiclassical Bekenstein-Hawking area law [42, 43] for the non-static FRW space-time at the apparent horizon. Also we shall calculate the corrections to the semiclassical entropy due to quantum effects of the Hawking like temperature. So we start with the thermodynamical law which express the energy conservation as

$$dM = T_h dS_A \tag{46}$$

Here T_h is the temperature of the horizon (with quantum corrections), S_A the entropy of the horizon and M is chosen as the Misner-Sharp gravitational mass, defined as [7]

$$dM = \frac{R_A}{2} \tag{47}$$

on the horizon.

Now if we identify M in the quantum corrections (28) for the temperature as the above Mishner-sharp mass then integrating (46) we have

$$\begin{aligned} S_A &= \int \frac{dM}{T_h} = \int \frac{4\pi M}{\hbar}\left[1 + \Sigma\alpha_k\left(\frac{\hbar}{M^2}\right)^k\right] dM \\ &= \frac{2\pi M^2}{\hbar} + 4\pi\alpha_1 \ln M - 4\pi\frac{\alpha_2\hbar}{M^2} + O\left(\hbar^2\right) \\ &= \frac{1}{2}\frac{A^2}{4\hbar} + 4\pi\alpha_1 \ln M - 16\pi\frac{\alpha_2\hbar}{R_A^2} + O\left(\hbar^2\right) \end{aligned} \tag{48}$$

In the above the first term on the r.h.s. is the semiclassical relation between area and entropy which is the usual Bekenstein-Hawking area law with a discrepancy of factor $\frac{1}{2}$. The second term is the leading order quantum correction term and is the standard logarithmic correction term in black hole thermodynamics [44, 45, 46, 47]. The higher order correction terms are in the inverse powers of the area of the horizon.

6 Summary

In this work, we have studied Hawking like radiation from the FRW model of the universe using both the approaches – the radial null geodesic method (tunnelling approach) and the HJ formalism. In both the methods we have obtained identical Hawking-like temperature at the semiclassical level. Also we have shown that choice of coordinate is not at all matter in HJ approach. The factor of $'two'$ problem in the tunnelling approach has been overcome considering Kodama vector instead of time-like vector and associated energy as the energy of the tunnelling particle. We have obtained quantum corrections to the Hawking-like temperature using HJ method and these corrections are similar to these for general static black holes. Also, quantum corrected entropy formula has been evaluated from the law of thermodynamics and it is found that usual entropy law with quantum corrections are same as in black hole thermodynamics with some discrepancies in the multiplicative factor. For future work we shall attempt to find interpretation of the parameters involved in the quantum corrections. It is worthy to generalize the tunnelling approach for non-zero mass particles(i.e., time-like geodesics) as well as to incorporate the quantum correction terms (back reaction effects). Finally, we shall try to resolve the ambiguity in the multiplicative factors for the entropy-area formula.

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